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IDENTIFICATION OF THE ELECTRICAL IMPEDANCE OF AN IMMOBILIZED TRANSDUCER AND THE MECHANICAL IMPEDANCE OF AN ELECTRODYNAMIC LOUDSPEAKER USING AUTOMATIC FIT OF THE FORCE FACTOR BL

Denys Volkov *
Vitaliy Didkovskyi *

**National Technical University of Ukraine "Igor Sikorsky Kyiv Polytechnic Institute", Faculty of Electronics, Peremohy Ave, 37, 03056, Kyiv, Ukraine*
e-mail: denys@volkov.in

***Abstract**: This paper describes a method for determining mechanical and electrical impedances of an electrodynamic transducer based on the automatic brute-force search of the force factor Bl. Also, the way of identifying electrical apparent resistance and apparent inductance of the voice coil is shown as an example of deeper loudspeaker analysis that proposed method allows to achieve. The measured signals are voltage on speaker terminals, voice coil current and membrane displacement. Measuring displacement in addition to voltage and current allows to completely separate transducer mechanical subsystem from electrical and apply various known modeling approaches to them separately.*

***Keywords:** electroacoustics, electrodynamic transducer, force factor Bl, brute-force search optimization, electromechanical analogies, complex impedance, apparent resistance, apparent inductance*

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1. INTRODUCTION

Nowadays the most widely used approach to model loudspeaker behavior at low frequencies is the lumped element modeling using electromechanical analogies. The basics of this approach consists in similarity of physical processes in electrical circuits and mechanical systems. In this way, it is possible to apply well known methods from the theory of electrical circuits to analyze electrical, mechanical and acoustical systems or their combinations.

The methods of loudspeaker modelling using lumped elements and electric equivalent circuits became widely used in the industry due to the work of Neville Thiele and Richard Small. They were also the first who proposed some practically convenient ways to determine loudspeaker parameters using added mass or added volume (Thiele, 1971; Small 1972). Together with growing computational capability, loudspeaker identification methods also became more complex and loudspeaker models more precise. Most of the modern and precise approaches to loudspeaker analysis requires direct measurements of transducer's mechanical parameters. For such purposes the most common ways are Doppler interferometry to measure loudspeaker membrane velocity (Moreno, 1991) or laser triangulation method to measure membrane displacement (Klippel, 2001).

The method presented in this paper also requires measurement of loudspeaker's membrane displacement together with voltage on speaker terminals and voice coil current. These three values allow to completely characterize electrical and mechanical subsystems of a loudspeaker (i.e. to determine mechanical and electrical impedances) and thus use various approaches to model them separately. The separation of mechanical and electrical subsystems is a big advantage of described method because it allows to treat and study them separately, avoiding any influence of mechanical parameters on electrical and vice versa.

2. BASIC DERIVATIONS

Let's consider a generalized model of an electrodynamic transducer without acoustical part (acoustical impedance is comprised into mechanical one) shown in the Fig.1:

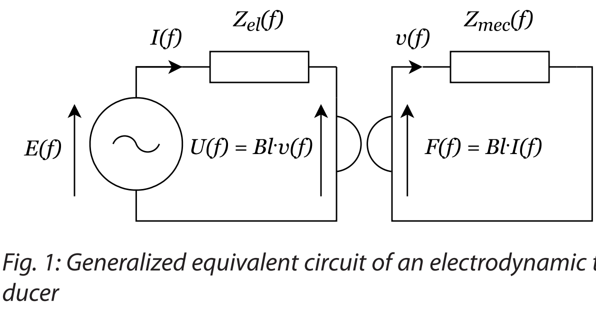

Fig. 1: Generalized equivalent circuit of an electrodynamic transducer

According to the theory of linear systems, both electrical subsystem on the left and mechanical subsystem on the right can be completely characterized by their complex impedances in frequency domain $\boldsymbol{Z_{el}(f)}$ and $\boldsymbol{Z_{mec}(f)}$ respectively. In a case of using the force analogy for the mechanical subsystem model, the gyrator component in between couples these subsystems through the force factor ***Bl***, and make them influence each other. This coupling complicates the loudspeaker analysis in cases when mechanical and electrical subsystems need to be studied separately. The main purpose of this paper is to demonstrate how mechanical and electrical subsystems can be identified completely decoupled from each other by applying a brute-force search method to identify the force factor ***Bl*** value.

In electrical domain, an equivalent loudspeaker circuit corresponds to Thevenin Circuit (voltage divider) as shown in the Fig. 2:

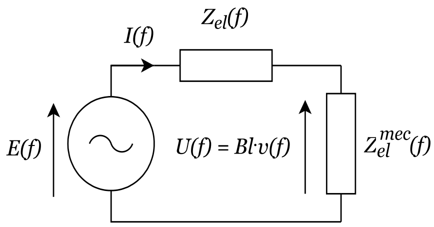

Fig. 2: Equivalent circuit of electrodynamic transducer in electrical domain

In this way, it is possible to write the equation for the total input electrical impedance of a transducer $Z_{tot}(f)$ as:

$$Z_{tot}(f) = Z_{el}(f) + Z_{el}^{mec}(f) \tag{1}$$

where:

$Z_{el}(f)$ - electrical impedance of an immobilized transducer,
$Z_{el}^{mec}(f)$ - electrical motional impedance

$$Z_{el}^{mec}(f) = \frac{U(f)}{I(f)} = \frac{Bl \cdot v(f)}{F(f)/Bl} = Bl^2\frac{v(f)}{F(f)} = \frac{Bl^2}{Z_{mec}(f)} \tag{2}$$

Total input electrical impedance of a transducer (equation 1) can be measured directly by using only electrical parameters. Classical methods for loudspeaker identification (Small, 972) only measure input voltage ***E(f)*** and voice coil current ***I(f)***, so to determine all speaker parameters some perturbation of mechanical system (adding mass or volume) and additional measurements are needed. Simultaneous measurement of a mechanical responses (membrane displacement or velocity) together with voltage and current allows to directly determine a state of mechanical subsystem and completely characterize a loudspeaker in one measurement without any physical perturbations.

Let's now express the voice coil electrical impedance through total input impedance and mechanical impedance:

$$Z_{el}(f) = Z_{tot}(f) - Z_{el}^{mec}(f) \tag{3}$$

And use measurable parameters in equation 3:

$$Z_{el}(f) = \frac{E(f)}{I(f)} - \frac{U(f)}{I(f)} = \frac{E(f)}{I(f)} - Bl\frac{v(f)}{I(f)} \tag{4}$$

As it can be seen, the right-hand side of equation 4 contains input voltage ***E(f)***, measured responses ***I(f)*** and ***v(f)*** and unknown parameter ***Bl***. After the force factor ***Bl*** is determined, the mechanical subsystem can be completely separated from the electrical, i.e. mechanical and electrical impedances $Z_{mec}(f)$ and $Z_{el}(f)$ are found separately.

3. BRUTE-FORCE SEARCH OF THE FORCE FACTOR BL VALUE

The first proposal to separate mechanical and electrical impedances by varying force factor Bl is done in the paper (Novak, 2019) by Antonin Novak. It refers to the known fact, that the resonance peak that can be seen in the total input electrical impedance $Z_{tot}(f)$ is due to the resonance phenomenon in mechanical subsystem $Z_{mec}(f)$ Electrical subsystem itself is not resonant and the electrical impedance of an immobilized transducer $Z_{el}(f)$ should gradually increase with frequency without resonances. Taking all stated above in account, it is possible to find such a value of the force factor ***Bl*** in equation 4 that resulting electrical impedance will be a smooth monotonically increasing function as it is required by physics.

In order to illustrate this concept, let us consider the Fig. 4. Presented curves were obtained using measured signals ***e(t)***, ***i(t)*** and ***x(t)*** on a typical 2-inch loudspeaker, where ***e(t)*** corresponds to the voltage over loudspeaker terminals, i***(t)*** – voice-coil current (measured thought 1 Ohm series resistor in the impedance box), ***x(t)*** - instantaneous displacement of loudspeaker's membrane that was captured with laser triangulation method using Keyence laser. All data acquisition was done using RME Fireface UC audio interface.

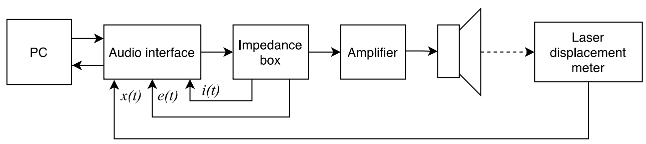

Fig. 3 Measurement setup

Measured time domain signals were transformed to the frequency domain, resulting in ***E(f)***, ***I(f)*** and ***v(f)*** signals respectively (taking in account, that in the linear approximation ***v(f)=j ωx-(f)*** membrane velocity)

To obtain curves in the Fig. 4, measured ***E(f)***, ***I(f)*** and ***v(f)*** signals were substituted into the equation 4 and **Bl** value was varying manually from 0 N./A (upper blue curve) to 3.5 N./A (lower magenta curve) with 0.5 N./A step.

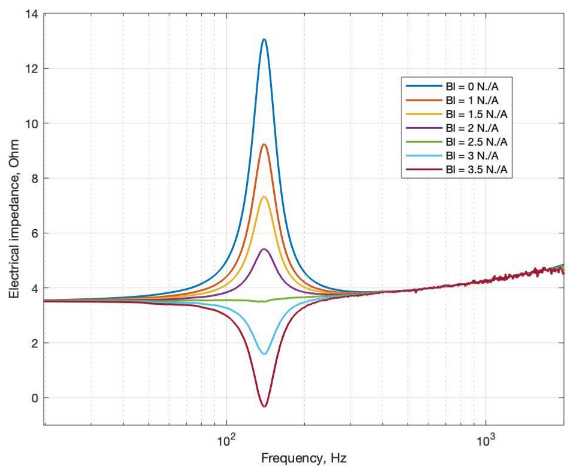

Fig. 4 Dependence of the calculated electrical impedance Zel(f) in equation 4 from the force factor value Bl

As it can be seen, for increasing values of ***Bl*** the resonance peak starts to decrease and after passing the optimal value it appears again. This illustrates that the value ***Bl = 2.5 N./A*** is very close to the actual force factor of the measured transducer.

Comparing to manual Bl adjustment (Novak, 2019), the method described in this paper allows to automatize and accelerate the identification procedure, determine ***Bl*** value more precisely, eliminate human factor and obtain reliant results repeatedly. It can be achieved by implementing simple optimization algorithm – one dimensional brute-force search over properly defined space $\boldsymbol{Bl \in (Bl_1, Bl_2)}$ and using appropriate cost function. Measurement frequency range should be large enough to capture mechanical resonance peak, but not limited to – much wider frequency range can be used without any manual adjustments of the algorithm.

It was discovered, that the convenient cost function for this method is a difference between the actually calculated $\boldsymbol{Z_{el}(Bl,f)}$ value from the equation 4 based on the measured data, and its approximated model $\boldsymbol{\widehat{Z_{el}}(Bl,f)}$. As it was described above, from the physical point of view, the voice coil electrical impedance $\boldsymbol{Z_{el}(Bl,f)}$ should be a smooth monotonically increasing function of frequency. That's why it can be successfully approximated with the second order polynomial function. In this case, when Bl value is far from optimal, the electrical impedance response $\boldsymbol{Z_{el}(Bl,f)}$ calculated from equation 4 will have a resonant peak that won't be captured by second order polynomial, thus the error between $\boldsymbol{Z_{el}(Bl,f)}$ and its polynomial fit $\boldsymbol{\widehat{Z_{el}}(Bl,f)}$ will be high. Closer to the optimal Bl value the resonant peak will disappear, so the second order polynomial function will better approximate measured response $\boldsymbol{Z_{el}(Bl,f)}$ and the error will be decreased. The electrical impedance modeled with second order polynomial function will be:

$$\widehat{Z_{el}}(Bl,f) = a(Bl)f^2 + b(Bl)f + c(Bl) \tag{5}$$

where:

a(Bl), b (Bl), c (Bl) - polynomial coefficients that provides the best fit of function for a particular ***Bl*** value in LMSE sense

More formally the cost function ***J(Bl)*** can be written as:

$$J(Bl) = E_f\left[\left|\widehat{Z_{el}}(Bl,f) - Z_{el}(Bl,f)\right|\right] =$$
$$= E_f\left[\left|\left(a(Bl)\cdot f^2 + b(Bl)\cdot f + c(Bl)\right) - \left(\frac{E(f)}{I(f)} - Bl\cdot\frac{v(f)}{I(f)}\right)\right|\right] \tag{6}$$

Where:

$E_f\left[\left|\widehat{Z_{el}}(Bl,f) - Z_{el}(Bl,f)\right|\right]$ - mean value of the modulus of approximation error over frequency.

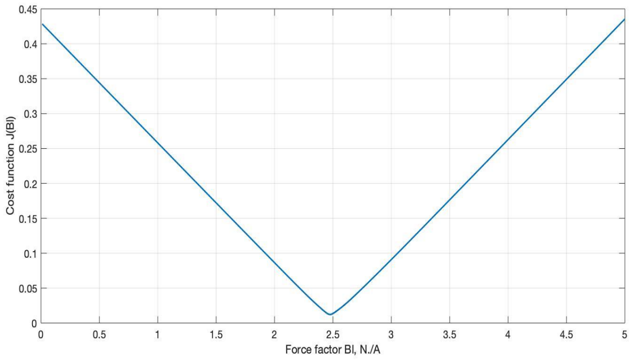

Fig. 5 Cost function J(Bl)

As it can be seen in the Fig.5, the cost function has a very well pronounced global minimum that confirms the appropriate choice of it (equation 6). In this way, minimization of $E_f[|\widehat{Z_{el}} - Z_{el}|]$ function allows to identify loudspeaker's force factor value Bl that is an important parameter for loudspeaker analysis:

$$\widehat{Bl} = \min_{Bl}[J(Bl)] \tag{6}$$

where:

$\boldsymbol{\widehat{Bl}}$ - optimal ***Bl*** value on the determined search space .

To summarize, the algorithm can be executed by following the next steps:

1. Measure frequency dependent responses of a loudspeaker: ***E(f)***, ***I(f)***, ***v(f)*** with reasonably long frequency vector ***f***
2. Define discrete search space $\boldsymbol{Bl \in (Bl_1, Bl_2)}$
3. For each Bl value from the search space calculate the electrical impedance $\boldsymbol{Z_{el}(Bl,f)}$ (equation 4) and find its second-order polynomial approximation $\boldsymbol{\widehat{Z_{el}}(Bl,f)}$ (equation 5)
4. Substitute these values into equation 6 and calculate the cost function ***J(Bl)***
5. Find the minimum value of the cost function and corresponding optimal $\boldsymbol{\widehat{Bl}}$

Knowing the force factor value, it is possible to completely decouple mechanical and electrical subsystems of a loudspeaker and work on their analysis separately, applying various models or identification techniques.

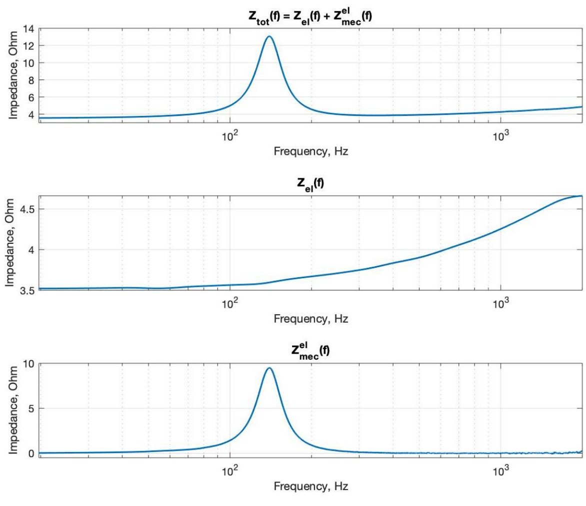

Fig. 6 Total input electrical impedance $\boldsymbol{Z_{tot}(f)}$ and its components: electrical impedance of an immobilized transducer $\boldsymbol{Z_{el}(f)}$, and electrical motional impedance $\boldsymbol{Z^{mec}{}_{el}(f)}$

Ones the force factor ***Bl*** and electrical motional impedance $Z^{mec}_{el}(f)$ are both identified, it's possible to determine the mechanical impedance of an electrodynamic loudspeaker as follow:

$$Z_{mec}(f) = \frac{Bl^2}{Z^{mec}_{el}(f)} \tag{8}$$

Frequency dependent behavior of the mechanical impedance $Z_{mec}(f)$ is illustrated below.

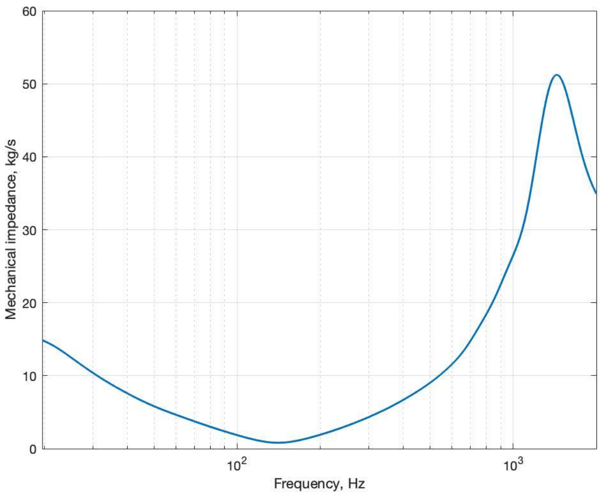

Fig. 7: Mechanical impedance of an electrodynamic loudspeaker

In order to demonstrate some advantages of having mechanical and electrical subsystems separated, let's take a closer look on the identified complex electrical impedance of immobilized transducer $Z_{el}(f)$ and try to explore its real and imaginary parts separately. Real part of the voice coil impedance corresponds to the sum of DC resistance R_{DC} and frequency dependent so called "apparent resistance" $R_{VC}(f)$. Imaginary part corresponds to reactive impedance caused by frequency dependent "apparent inductance" (Novak,2019) $L_{VC}(f)$:

$$Z_{el}(f) = R_{DC} + R_{VC}(f) + j \cdot 2\pi f \cdot L_{VC}(f) \tag{9}$$

Where:

$$R_{DC} + R_{VC}(f) = Real[Z_{el}(f)]$$
$$L_{VC}(f) = \frac{Imag[Z_{el}(f)]}{2\pi f} \tag{10}$$

Note, that we still didn't imply any specific model of the voice coil and used only the data that can be retrieved from the actual measurements.

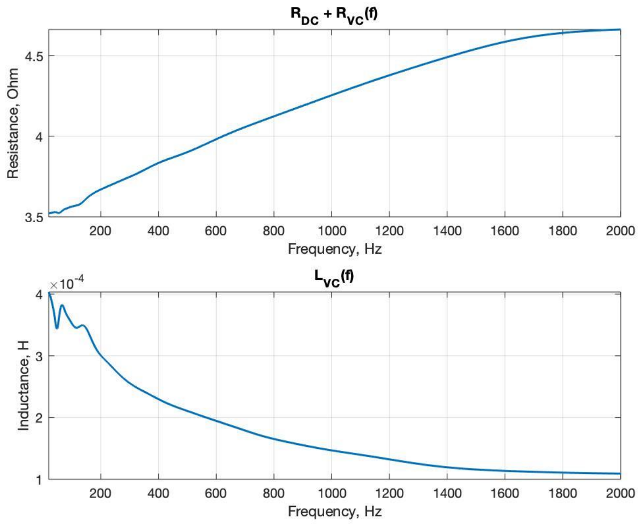

Fig. 8: Frequency dependent apparent resistance and apparent inductance of the voice-coil

The Fig. 8 shows the actual quite complicated frequency behavior of voice coil that includes the effect of eddy currents, thermal influence, skin effect, etc. These curves can be further modeled using lumped element single values, different types of curve fitting and other methods such as (Vanderkooy, 989) or (Wright, 1990), depending on the target application and desired precision level.

Despite the main aim of the paper is to introduce the automated ***Bl*** identification approach for loudspeaker analysis, this brief example above shows how precise and efficient the process of modeling and analysis can be when electrical impedance is separated from mechanical one. This is impossible to achieve with classical measurement approaches using added mass or volume, or based on curve fitting methods. Furthermore, much deeper analysis of the moving part can be perform using identified complex mechanical impedance, but that is not in the scope of this paper.

4. CONCLUSION

This paper has described a method for automated identification of loudspeaker's force factor ***Bl*** based on measured voice coil current, input voltage and membrane displacement. Using identified force factor, it is possible to completely separate complex mechanical impedance of a moving part from complex electrical impedance of the voice coil. Ones both impedances are separated it is possible to apply any analysis or modeling approaches to them separately avoiding mutual influence of mechanical parameters on electrical and vice versa. As an example of more sophisticated analysis, the way of determining voice coil apparent resistance and apparent inductance from measured complex electrical impedance is shown. Described method can be applied to various kinds of moving coil loudspeakers including microspeakers and fragile tweeters, unlike classical approaches with added mass or added volume. Comparing to the other methods that use displacement or velocity measurement, brute-force ***Bl*** optimization allows to analyze loudspeakers much deeper and increase modelling precision. Comparing to manual ***Bl*** adjustment introduced in (Novak, 2019), brute-force search allows to speed up and automize the identification process, achieve more precise results with higher reliability as human factor will be excluded from the measurement.

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