SOUND REFLECTIONS

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ABSTRAKT

The present paper is a review of the author's work on sound reflections covering a span of years starting from 1982. After a study of the reflection properties of curved surfaces, the diffraction effects related to single reflectors was studied, both theoretically and experimentally. The main result was a characteristic frequency above which the sound reflection is fully efficient as long as the reflection point is within the reflecting surface. Next, the array of reflectors was studied, and again the characteristic frequency was found useful as a design parameter, but here as an upper frequency limit for reliable reflections. The design principle was successfully applied in a concert hall in Copenhagen. Recently, the reflection of a spherical sound wave emitted from a point source has been studied and a theoretical model has been developed. An application of this model is for the sound propagation over an area with sound absorbing properties like those of a seated audience in a large auditorium.

1. INTRODUCTION

A general model of sound reflection from a surface is shown in Figure 1.

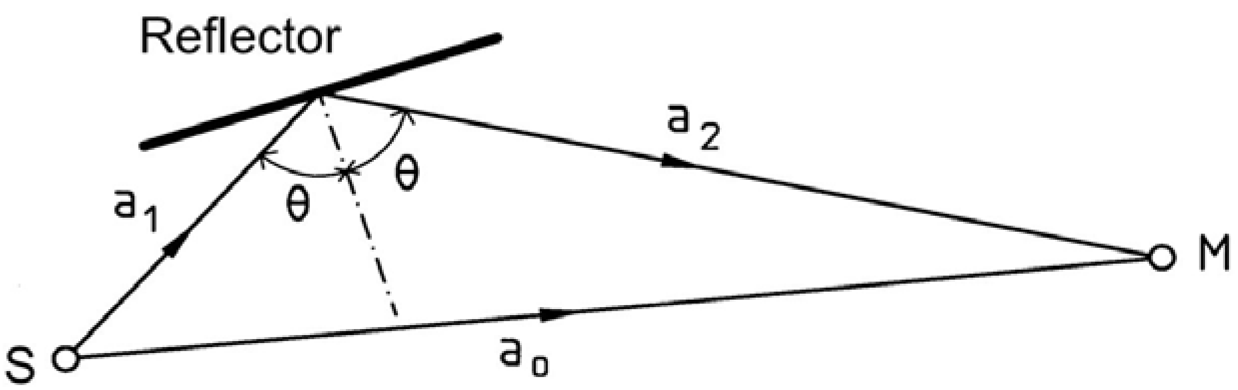

Figure 1: Distances and angles related to the sound reflection from a surface. Sound is emitted from a point source (S) and the receiver point is M.

The attenuation of the reflected sound relative to the direct sound is:

$$\Delta L = L_{refl} - L_{dir} = \Delta L_a + \Delta L_m + \Delta L_k + \Delta L_s \quad \text{(dB)} \quad (1)$$

where the four attenuation terms are related to distance (a), absorption due to the material (m), curvature of the surface (k), and finite size of reflector (s). The attenuation terms are usually negative corresponding to the reflected sound pressure level being lower than the direct sound pressure level.

The attenuation due to distance DL_a is -6 dB per doubling of the distance due to the energy spread on the surface of a sphere:

$$\Delta L \quad = 20 \lg \frac{\quad}{(\quad)} \qquad \text{(dB)} \tag{2}$$

If the angle-dependent absorption coefficient of the material is α_θ, the reflected sound energy is $(1 - \alpha_\theta)$ times the incident sound energy, and the attenuation due to the material DL_m is:

$$\Delta L \quad = 10 \lg(1 - \alpha\,) \qquad \text{(dB} \tag{3}$$

The characteristic distance a* is defined as the harmonic average of the two distances from the reflection point to the source (a_1) and to the receiver (a_2):

$$a^* = \frac{\quad}{(4)} \tag{4}$$

2. REFLECTION FROM A CURVED SURFACE

From a geometrical analysis of the reflection of a sound wave from a curved surface with radius R, the attenuation due to curvature is found to be [1]:

$$\Delta L \quad = -10 \lg \; 1 + \frac{a^*}{R \cos\theta} \qquad \text{(dB)} \tag{5}$$

By convention R is negative for a concave surface and positive for a convex surface. The formula (5) is illustrated in Figure 2, and it is noted that the sound is always attenuated when reflected from a convex surface. However, in case of a concave surface, the sound pressure level can be increased due to a focusing effect.

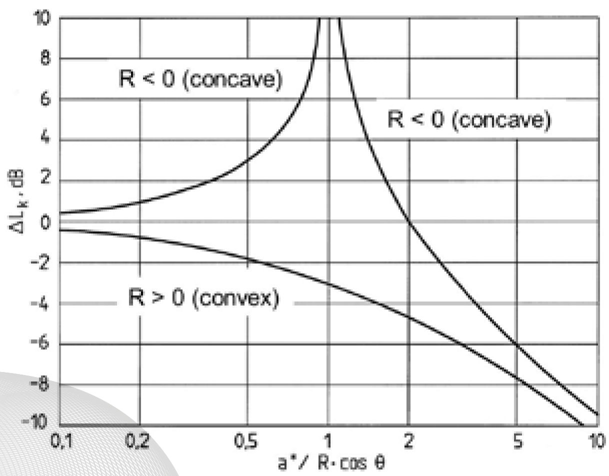

Figure 2: The attenuation due to curvature.

3. SINGLE REFLECTORS AND DIFFRACTION EFFECTS

The attenuation of a sound reflection from a rectangular plate can be calculated by the Kirchhoff-Fresnel approximation as presented by the author in 1986 [2]. The solution implies a transformation of variables into u and v, which are orthogonal directions on the surface of the plate, see Figure 3. The origo of this coordinate system is the geometrical reflection point. In one direction, the variable is u:

$$u = \sqrt{\frac{2}{\lambda}\left(\frac{1}{a} + \frac{1}{a}\right)} \cdot \xi \tag{6}$$

where λ is the wavelength, a_1 and a_2 are the distances (see Figure 1) and ξ is the distance from the reflection point. So, if the width of the plate is 2b and the distance from the reflection point on the plate to the nearest edge is e, we have for u_1 that ξ = e cos θ and for u_2 that ξ = (2b – e) cos θ.

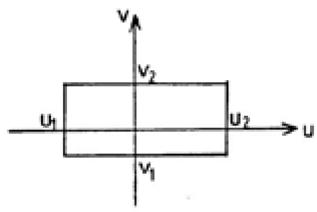

Figure 3: The parameters u_1, u_2, v_1, and v_2 that correspond to distances from the reflection point to the edges of the reflecting plate.

The attenuation of the reflection due to diffraction is:

$$\Delta L \quad = 10 \lg(K \cdot K\,) \qquad \text{(dB)} \tag{7}$$

where the coefficients K_1 and K_2 represents the two orthogonal directions of the plate. They are calculated by means of the Fresnel integrals C(x) and S(x), and for the first direction we get:

$$K = \_ \qquad C(u\,) - C(u\,) \qquad + S(u\,) - S(u\,) \tag{8}$$

and similarly for K_2 with the v_1 and v_2 parameters.

The result in Eqn. (8) can be visualised with the Cornu's spiral, which is the graph created when C(x) and S(x) are displayed as abscissa and ordinate, respectively, see Figure 4 a. An example of a low frequency is $u_2 = -u_1 = 0.7$ (red arrow in Figure 4). The length of the arrow is proportional to the amplitude of the sound pressure in the reflected sound wave. A very high frequency or infinitely large plate is represented by $u_2 = -u_1 \rightarrow \infty$. This yields the coefficient $K_1 = 1$, and thus the attenuation $DL_s = 0$ dB (blue arrow in Figure 4).

The results show that the sound is reflected efficiently at high frequencies althought with some fluctuations, but the sound is attenuated below a certain frequency. This led to the introduction af the characteristic frequency f_g for a single reflector:

$$f = \frac{c\, a^*}{2\, S \cos\theta} \quad \text{(Hz)} \tag{9}$$

Measurements in the large anechoic chamber at DTU were made using an impulse gating technique [3]. The set-up is seen in Figure 5. and an example of the measurement results are seen in Figure 6. The agreement between the measurement results and the calculations with the Kirchhoff-Fresnel approximation is in general very good. Only, at frequencies below c. 125 Hz the measurements deviate from the predictions, probably due to limitations in the measurement technique.

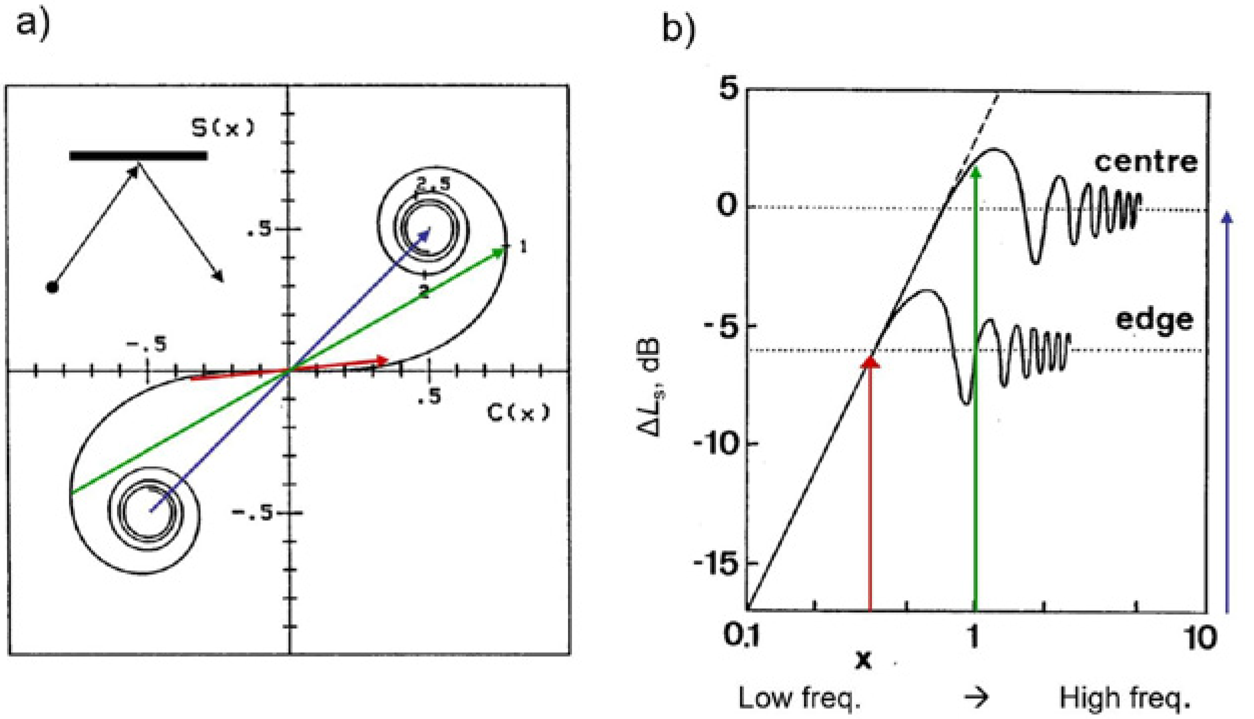

Figure 4: Attenuation due to diffraction of sound reflection from a plate. a) The Cornu's spiral and reflection point in the centre of the plate; low frequency (red arrow), middle frequency (green arrow), high frequency limit (blue arrow). b) Attenuation in dB for reflection point in centre or on the edge.

Figure 5: Set-up in anechoic chamber foe measurement of sound reflection from a single reflecting plate.

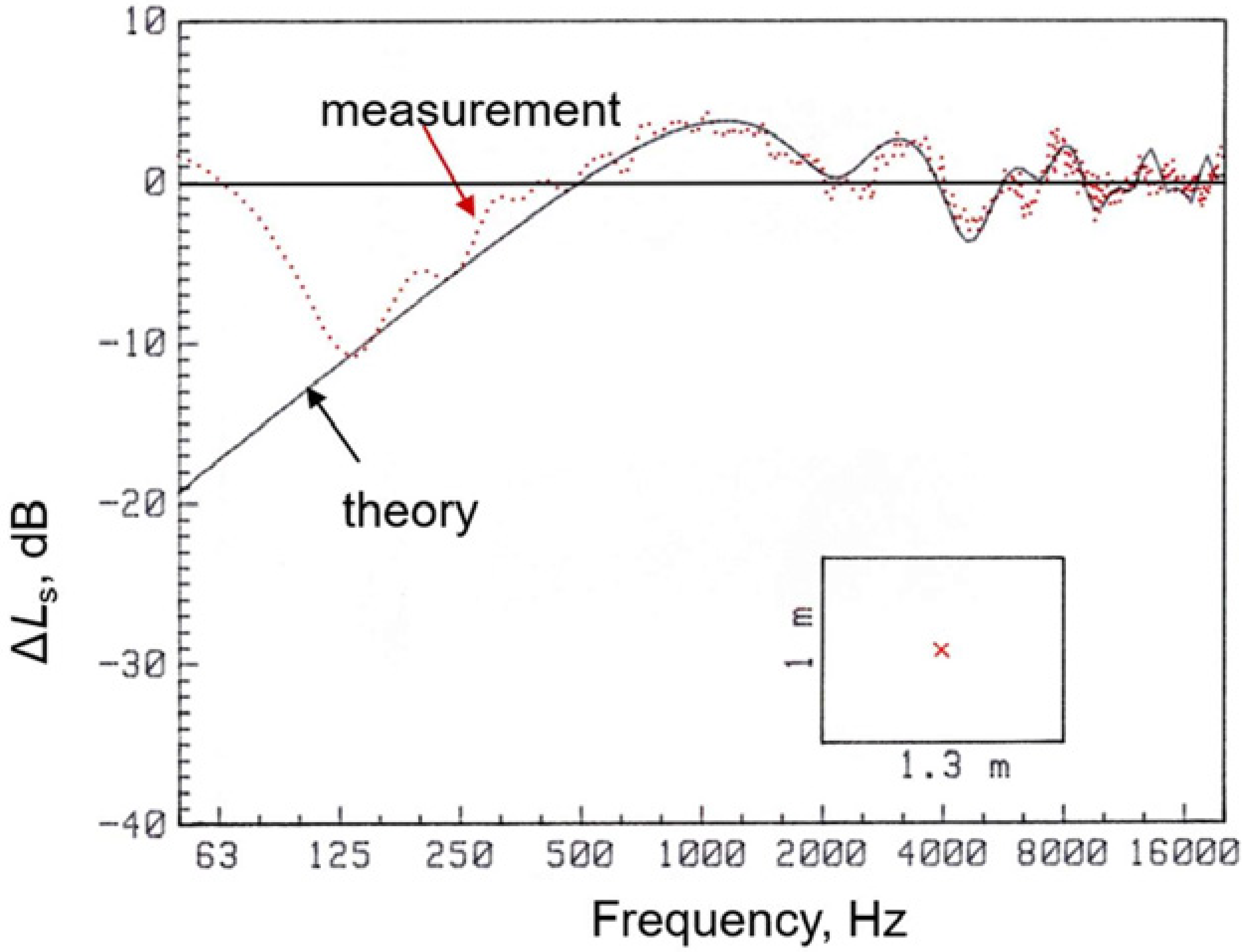

Figure 6: Measured and calculated attenuation of sound reflection from a rectangular plate. Angle of incidence θ = 30°, a1 = a2 = 3.0 m. Adapted from [3].

The conclusion from the study of single reflectors is that the characteristic frequency f_g should be as low as possible in order to reflect a wide frequency range. This means that the reflector plates should be as large as possible.

4. ARRAY OF REFLECTORS

The study of sound reflections was continued with investigations on arrays of reflectors. The theoretical model based on the Kirchhoff-Fresnel approximation was extended to a two-dimensional pattern of rectangular reflector plates, see Figure 7 and 8.

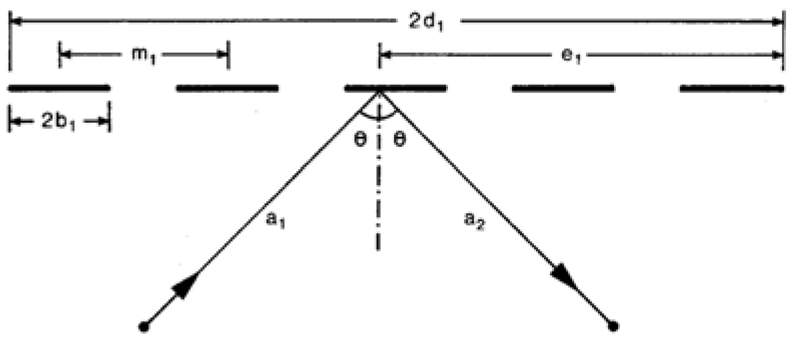

Figure 7: A reflector array with five rows of reflector plates. In incident and reflected sound are indicated as rays.

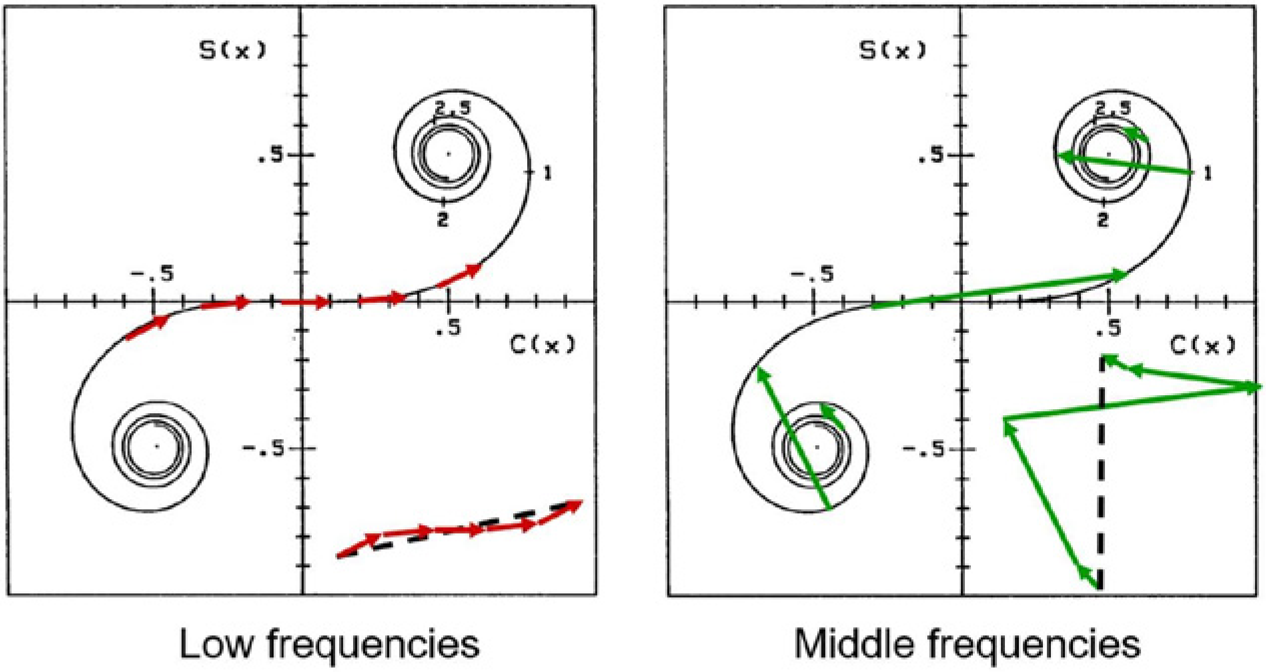

Figure 8: Application of the Cornu's spiral for a reflector array with five rows as in Figure 7. At low frequencies the contributions from each row are approximately in phase. At middle frequencies the contributions add like vectors due to different phase.

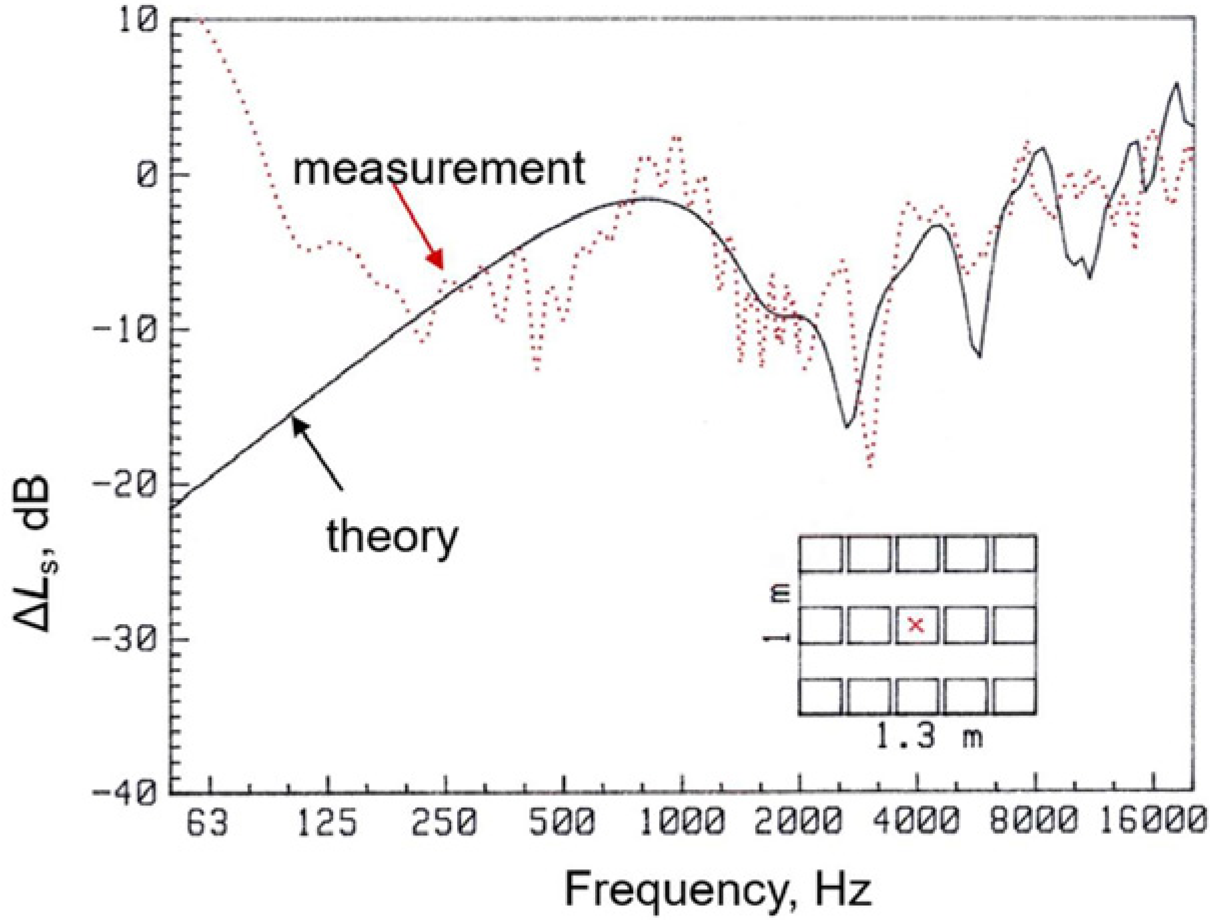

*Figure 9: Measured and calculated attenuation of sound reflection from a reflector array. Plate dimensions 0.23 m * 0.20 m. Angle of incidence θ = 30°, a1 = 3.0 m, a2 = 2.0 m. Adapted from [3].*

The theoretical model for the reflector array can explain that all the plates work together at low frequencies, and the result is comparable to that for a very large reflector. However, for middle and high frequencies the reflection is dominated by that from the particular plate on which the geometrical reflection point is located. The other plates contribute less and with different phase, see Figure 8. The theoretical model was compared with measurements, and one example is seen in Figure 9.

Further analysis of reflector arrays was presented in 1990 [4]. The calculation results were compared with the characteristic frequency for a single plate, Eqn. (9) and one example is seen in Figure 10. Above the characteristic frequency the results vary depending on the exact position of the reflection point. If the reflection point is located between the plates, the high frequency reflection is weak and rapidly decreasing with frequency. Below the characteristic frequency, the results are quite stable and the position of the reflection point is not so important. Actually, the low frequencies are reflected quite well, and the level has been shown to depend on the density, i.e. how close the plates are together in the array.

It was found that the characteristic frequency f_g for the plates in the reflector array is also relevant for a reflector panel, but with a very different meaning. While f_g shall be as low as possible for a single reflector, the opposite is true for a reflector array. The best and most even reflection over a wide frequency range is obtained with a reflector array consisting of many small plates. At frequencies above f_g the reflections can vary significantly with small movements of the receiver position, see Figure 10.

The design principle was applied for the acoustical refurbishment of the concert hall in Copenhagen, originally used by the Danish Radio, but today taken over by the Royal Academy of Music. As seen in Figure 11, 61 slightly bent reflector plates are arranged in five rows, and each reflector has the dimensions 1.8 m * 0.8 m. With $a_1 = a_2 = 7.07$ m and θ = 45°, the characteristic frequency is $f_g = 1200$ Hz. More details can be found in [5].

Unfortunately, this design principle for a reflector array is seldomly followed. In several newly designed concert halls it is seen that the reflector array over the orchestra podium is designed with a few, very large reflectors. It seams like the fundamental difference between single reflectors and a reflector array is not understood.

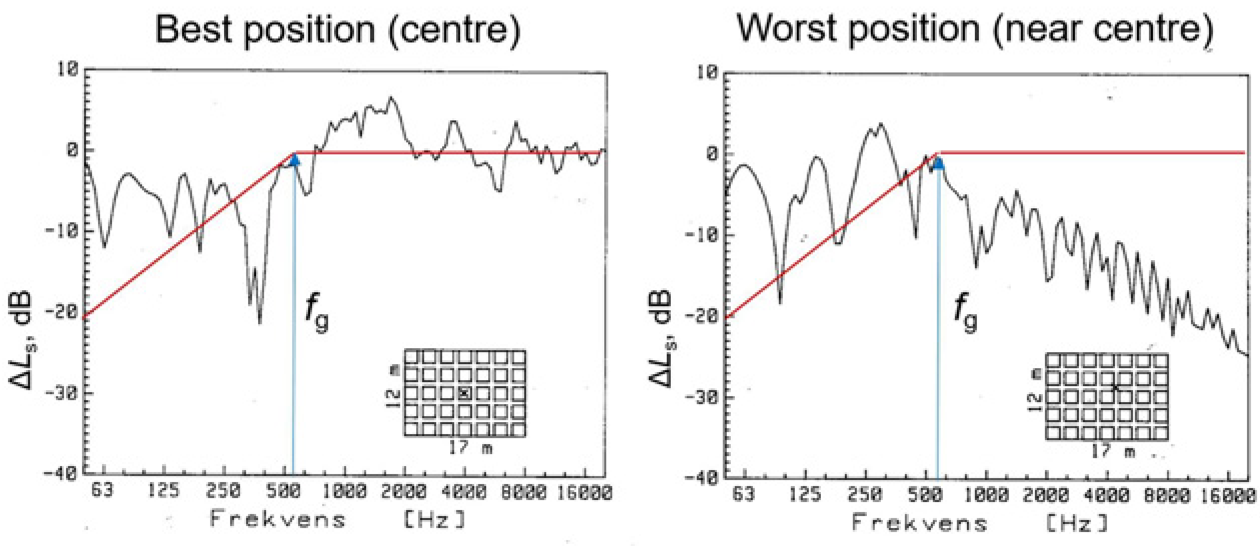

*Figure 10: Calculation results for a reflector panel. Left with reflection point in centre of the middle plate. Right with the reflection point between plates, but near the middle of the array. Plate dimension 1.8 m * 1.8 m. Density of array = 50 %, Angle of incidence θ = 30°, a1 = a2 = 7.1 m. The characteristic frequency for a single plate is indicated. Adapted from [4].*

Figure 11: Photo from the concert hall after the new reflector array and new wall reflectors were installed in 1989.

5. REFLECTION OF SOUND IN NON-SPECULAR DIRECTION

The sound reflection from a rectangular plate was further studied in 2004 by the application of a result derived for the sound transmission though a rectangular opening [6]. The directional sound radiation was averaged over frequency bands of one octave. The frequency dependent parameter is ka, where k is the angular wave number and a is one-half the side of a square with the same area as the plate. An example of calculated sound reflection is shown in Figure 12 for the angle of incidence 60° relative to the normal of the plate. While the sound reflection is within a narrow wedge for ka high, the reflection is broadening into a wider fan of directions when ka decreases. For ka = ¼ the reflection is nearly omnidirectional within a half-sphere. For comparison is shown in dotted line the Lambert's law for a diffuse reflection.

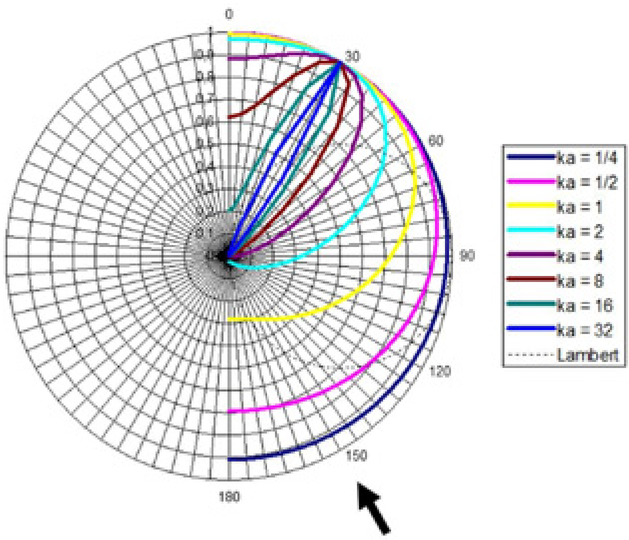

Figure 12: Directivity of sound reflection in octave bands for a plane sound wave incident in the direction indicated by the arrow. The reflecting plane is in the vertical axis (0 to 180 degrees). The frequency parameter is ka.

6. SPHERICAL WAVE REFLECTION

Sound reflections are usually calculated with the assumption that the incident sound is a plane wave, even if the source is a point source in a well-defined distance from the reflecting surface. It has been known for a long time (at least since 1951) that a spherical sound wave gives reflections that can deviate significantly from those generated by a plane wave. This has been studied for decades in connection with outdoor sound propagation, but still the spherical wave reflection is seldomly applied in room acoustics. One reason for this might be that the mathematics involved is quite heavy. Also, solutions found in the literature mostly assume that the reflecting surface has infinite area.

Figure 13 is an example of the sound emitted from a point source in a height h_s above a surface. Using the image source method, the reflected sound can be treated as the sound emitted from the image source and transmitted through the surface to the receiver point. The total sound pressure in the receiver point is the sum of the sound pressures of the two sound waves. The result is strongly dependent on the phase difference between the two sound pressures, and this is not only due to the different length of propagation, but also due to the phase shift at the reflection.

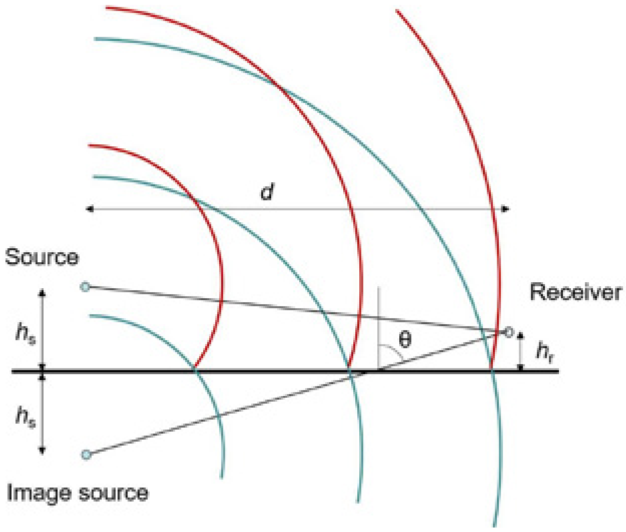

Figure 13: Spherical waves emitted from a point source and an image source. The receiver is a point in the height hr above the reflecting surface.

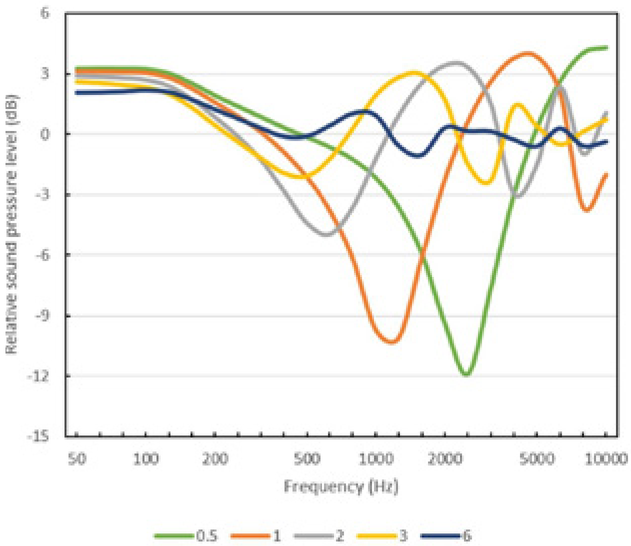

Figure 14: The sound pressure level relative to the direct sound, calculated with different height hs of the source over the reflecting surface (0.5 m to 6 m). The surface material has acoustical properties typical for audience on upholstered seats. Horizontal distance is fixed at d = 20 m and height of receiver above the surface is hr = 0.4 m. Area of reflecting surface is 1000 m².

Recently, the author has derived an approximate solution that includes the complex radiation impedance of a surface with finite area [7]. An example of application of this method is seen in Figure 14. The reflecting surface has acoustical properties typical for an audience on upholstered seats. The seated audience is modelled as a large box with the height 0.8 m. The height of the receiver (the ear of a seated person) is assumed to be 1.2 m above the floor, and thus h_r = 0.4 m. The height of the sound source is varied while the horizontal distance to the receiver is d = 20 m. The results show a pronounced dip in the frequency curve when the height of the source is low (below 3 m). But at the source height 6 m the dip is no longer present and the frequency curve is relatively flat. At frequencies below 200 Hz the sound pressure level is around 2 dB to 3 dB above that in a free field (direct sound alone).

In the future it shall be interesting to see if the spherical wave reflection will be better understood in general, and to what extend room acoustic computer models can benefit from that.

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