

COCHLEA FREQUENCY-POSITION FUNCTION OBTAINED FROM ATTENUATION CHARACTERISTICS OF BASILAR MEMBRANE WAVES

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Abstract: This study focuses on the attenuation characteristics of the wave that propagates on the basilar membrane. The basilar membrane wave, which occurs from the interaction between the basilar membrane and fluid, plays a major role in the hearing system. Because the mass and stiffness of the basilar membrane are graded along its length, the basilar membrane wave peaks at a specific region along the cochlea depending on the acoustic wave frequency. We took the viscosity of the fluid into consideration and investigated the wave characteristics by conducting modal analysis. We found that the attenuation curve of the basilar membrane wave has an inflection point and the frequency-position function obtained from the inflection points matches well with the Greenwood function. This suggests that the mode of the basilar membrane wave drastically changes before and after the peak point.

Keywords: Cochlea, basilar membrane, modal analysis, attenuation characteristics, frequency-position function

1. INTRODUCTION

The elastic wave that occurs from the interaction between the basilar membrane (BM) and fluid plays a major role in the hearing system. We call this wave the BM wave. The wave propagates along the cochlea from the base to the apex. Because the mass and stiffness of the BM are graded along its length, the elastic wave peaks at a specific region along the cochlea, depending on the acoustic wave frequency. Greenwood developed the cochlea frequency-position function (Greenwood function) based on physiological data. He integrated an exponential function fitted to a subset of frequency-resolution-integration estimates [1].

The vibration mechanics of the BM have been investigated using various numerical models [2–6]. Many researchers have used the Wentzel-Kramers-Brillouin (WKB) method, which is based on the wavenumber distribution along the cochlea [7–12]. This method inherently includes assumptions such as the wave propagation being limited to one direction and the variation of the wavenumber being small enough compared to the wavelength. The finite-element (FE) method has also been adopted, especially for 3D cochlea structures

[13–16]. Gan et. al. conducted an acoustic-structure-fluid coupled FE analysis on a human ear model consisting of the ear canal, middle ear, and cochlea [14]. Böhnke et. al., on the other hand, analyzed the fluid-structure couplings between the perilymph and the stapes footplate, round window membrane, and elastic cochlea partition using the FE method [13]. On the other hand, we have conducted elastic-wave modal analyses of the cochlea [17–19]. By applying the modal analysis to the cochlea, we can discuss which modes can be assisted by the structure that includes several membranes such as the BM, Reissner's membrane, and tectorial membrane. We can also illustrate the mode field and displacement of the membranes caused by the mode.

The human cochlea contains two types of fluid: perilymph and endolymph. Perilymph fills the scala vestibuli and scala tympani, while Endolymph fills the scala media. Both fluids have viscosity, which causes attenuation of the waves that occur from the interaction between the membranes and fluid. In this study, we focused on the attenuation characteristics of the BM wave and considered the viscosity of the fluid. We investigated the BM wave using modal analysis and discussed the relationship between the attenuation charac-

teristics and the Greenwood function. We employed COMSOL Multiphysics, based on the FE method, for our analysis.

2. MATERIALS AND METHODS

The human cochlea consists of a triple chambered duct that is divided by the BM and Reissner's membrane. We modeled it as a dual-chambered duct composed of the scala vestibuli and scala tympani. In this study, we focus on the influence of the BM; therefore, Reissner's membrane, the organ of Corti, and the inner and outer hair cells were not included in the analysis. Fig. 1 shows the analysis model.

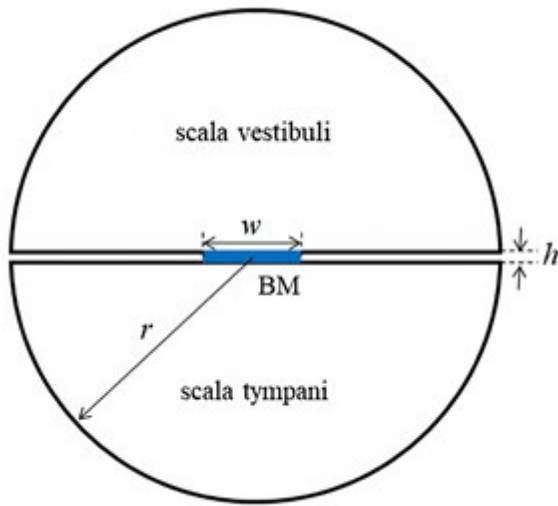


Fig. 1: Analysis model

The propagation characteristics, including the attenuation characteristics, were investigated by conducting modal analysis on the structure shown in this figure. The rigid boundaries were used to enclose the scala vestibule and scala tympani, aside from the BM. The cochlea duct had a circular shape, and the radius was uniformly 0.5 mm along the cochlea for the sake of simplicity. The width w , height h , and Young's modulus E of the BM varied along the cochlea in accordance with the following equations [14].

$$w = 0.1 [\text{mm}] + \frac{0.4 [\text{mm}]}{35 [\text{mm}]} \times z, \quad (1)$$

$$h = 7.5 [\mu\text{m}] - \frac{5 [\mu\text{m}]}{35 [\text{mm}]} \times z, \quad (2)$$

$$E = 50 [\text{MPa}] - \frac{47 [\text{MPa}]}{35 [\text{mm}]} \times z, \quad (3)$$

The length z along the cochlea varied from 0 to 35 mm. The viscosity, bulk modulus, and density of the fluid were 2.8×10^{-3} Pas, 2.2×10^9 Pa, and 1.034×10^3 kg/m³, respectively. The Poisson's ratio and density of the BM were 0.49 and 1.2×10^3 kg/m³, respectively [20, 21].

3. RESULTS AND DISCUSSION

In this chapter, all results were obtained through modal analysis using COMSOL Multiphysics, based on the FE method.

Fig. 2 shows the frequency characteristics of the propagation constants of the BM wave. The z along the cochlea was 10 mm, and the solid and dashed lines represent the phase and attenuation constants, respectively. The phase constant monotonically increases as the frequency becomes higher, indicating that the high frequency waves propagate slower than the low frequency waves. The velocity of the BM wave differed significantly depending on the acoustic frequency, with the attenuation constant sharply increasing around 5 kHz and maintaining a relatively constant value above this frequency. We then further examined the inflection point of the attenuation constant curve.

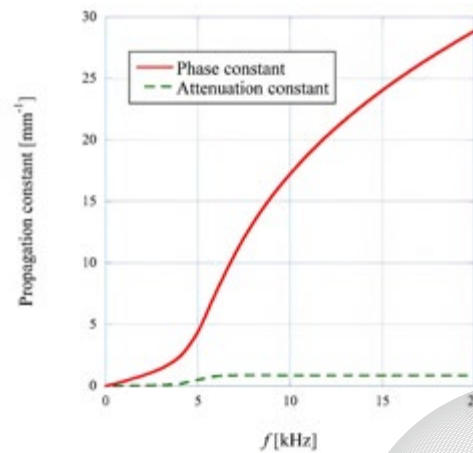


Fig. 2: Dispersion diagram of BM wave when $z=10$ mm

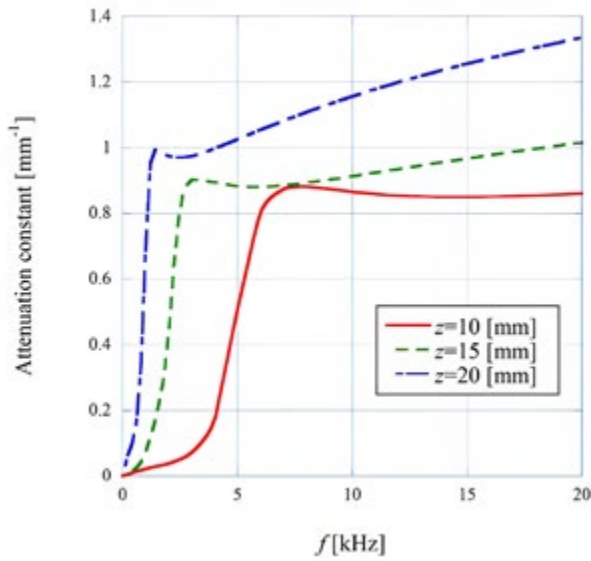


Fig. 3: Frequency characteristics of attenuation constants of BM wave when $z=10, 15$, and 20 mm

The frequency characteristics of the attenuation constants of the BM wave are shown in Fig. 3. The solid, dashed, and dash-dotted lines represent the results when z equals 10, 15, and 20 mm, respectively. The attenuation constants exhibited an inflection point at all z positions along the cochlea, and the frequency of the inflection point varied depending on z . For instance, when $z=20$ mm, the inflection point occurred at approximately 1.4 kHz, while it was about 3 kHz when z equaled 15 mm. It can be observed that the frequency of the inflection point increases as z decreases. This trend is similar to the cochlea frequency-position function.

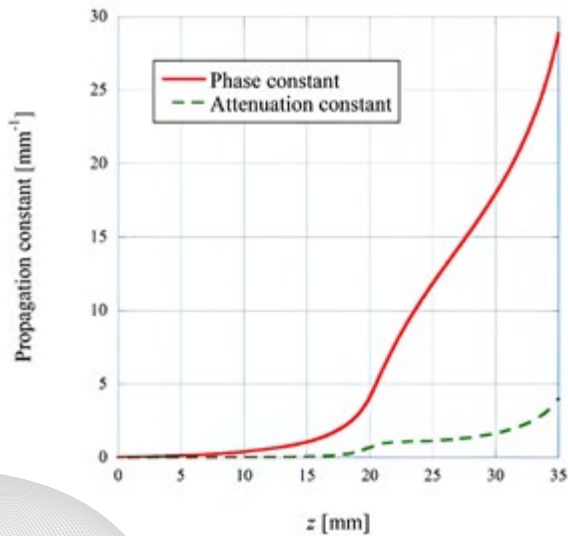


Fig. 4: Propagation constants of BM wave as a function of z when $f=1$ kHz

Fig. 4 illustrates the propagation constants of the BM wave as a function of z along the cochlea when f equals 1 kHz. The solid and dashed lines indicate the phase and attenuation constants, respectively. The phase constant exhibited a sharp increase when z exceeded approximately 20 mm. The attenuation constant also increased in the same region. Thus, we can conclude that the wave dissipates around this point and hardly propagates further toward the apex. Additionally, the attenuation constant curve exhibited an inflection point at approximately 22 mm.

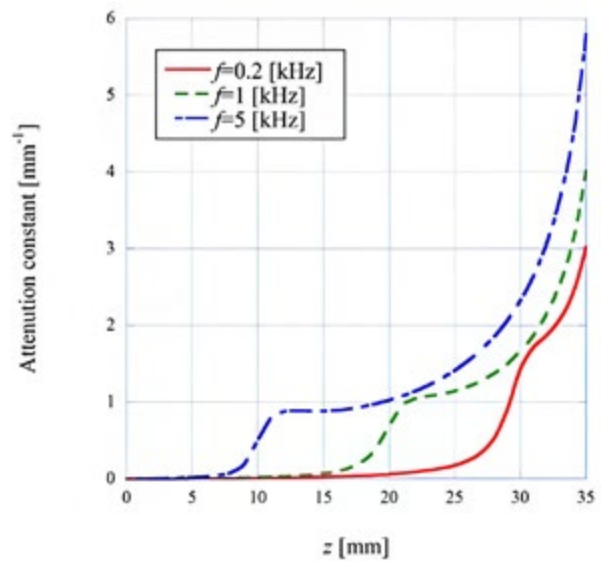


Fig. 5: Attenuation constants of BM wave as a function of z when $f=0.2, 1$, and 5 kHz

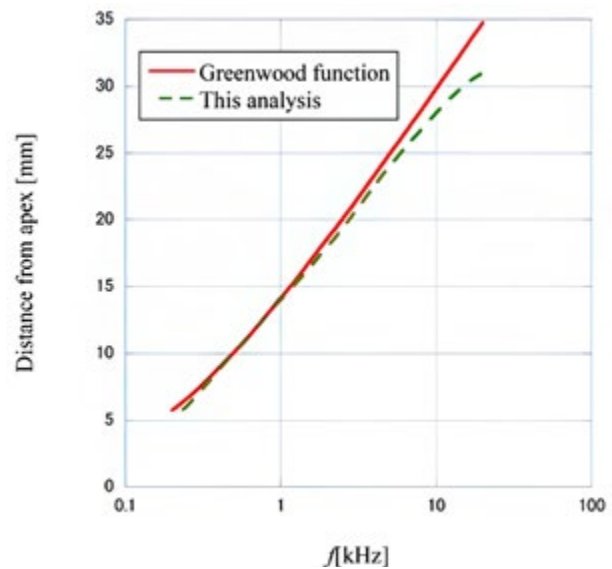


Fig. 6: Cochlea frequency-position function

Fig. 5 shows the attenuation constants of the BM wave as a function of z along the cochlea. The solid, dashed, and dash-dotted lines represent the results when f equals 0.2, 1, and 5 kHz, respectively. The attenuation curve as a function of z also shows an inflection point. For instance, the inflection point was about 11 mm when $f=5$ kHz and it increased as the frequency decreased. We then compared the frequency-position function obtained from the inflection points of the attenuation curve with the Greenwood function.

Fig. 6 shows the cochlea frequency-position function. The solid line represents the results calculated using the Greenwood function, which is expressed as follows [1]:

$$f = A\{10^{ax} - k\}, \quad (4)$$

where x [mm] is the distance from the apex, and the parameters were chosen as $A=165.4$, $a=0.06$, and $k=0.88$, which are the values for human ears. The dashed line shows the results obtained from the inflection points of the attenuation curve of the BM wave. The

results obtained from the attenuation curve have similar characteristics as the Greenwood-function results. The two curves match well in the low-frequency region. The BM wave with a certain frequency peaks at a specific region along the cochlea. At that region, the BM wave attenuation curve has an inflection point, suggesting that the mode of the BM wave undergoes a drastic change before and after the peak point.

4. CONCLUSION

We investigated the attenuation characteristics of the wave that propagates on the BM. We found that the attenuation curve of the BM wave has an inflection point, and that the frequency-position function obtained from the inflection points matches well with the Greenwood function. This suggests that the mode of the basilar membrane wave undergoes a drastic change before and after the peak point.

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